7. The California visibility standard requires that the visibility be greater than 10 miles on days when the relative humidity is less than 70%. Consider a day when the visibility controlling aerosol is composed of material with a refractive index of 1.5. Estimate the aerosol mass concentration ($\mu g/m^3$) in the atmosphere that would correspond to the visibility standard. Assume (1) the density of the spherical particles is 1 g/cm$^3$; (2) the aerosol is monodisperse with a particle size, $d_p = 0.5 \mu m$; and (3) the wavelength of interest is 0.5 $\mu m$. (8%)

\[
\rho = \frac{d}{\rho_p} = \frac{0.5 \times 10^{-6}}{1 \times 10^{-3}} = 5 \times 10^{-4} m
\]

\[
N = \frac{4 \pi d^3}{3 \rho \lambda} = \frac{4 \pi (0.5 \times 10^{-6})^3}{3 \times 10^{-4} m \times 5 \times 10^{-4} m} = 3.44 \times 10^8 \text{ cm}^{-3}
\]

\[
\sigma_e = 9.6 \text{ cm}^{-1} \text{ Figure 6.2}
\]

\[
N = 4 \pi d^3 \rho \sigma_e = 4 \pi (0.5 \times 10^{-6})^3 \times 10^{-4} \times 9.6 = 2.43 \times 10^4 \text{ cm}^{-3}
\]

\[
\sigma_e = \frac{N \pi d^3 \rho}{4}
\]

\[
C_m = 12.1 \mu g/m^3
\]

\[
C_m = 2 \pi d \sigma_e = \frac{2 \pi \times 0.5 \times 10^{-6} \times 5 \times 10^{-4}}{3 \times 9.6} = 2.25 \times 10^{-3} \mu g/m^3
\]
8. (a) What is the maximum distance at which one can see a black automobile on a foggy, moonlit night? Assume that the fog consists of 5 µm droplets at $10^{10}$ #/m$^3$, that the background luminance is $10^{-3}$ cd/m$^2$, and that your eyes are completely dark adapted. (b) Repeat part (a) if the headlights are on and are directed toward you; assume the inherent contrast is $10^4$. (10%)
Go over the “Optical Particle Counter” module at http://aerosol.ees.ufl.edu. Learn how to operate the OPC simulator. Use it to simulate the measurement of “room air” and outdoor air in St. Louis. Present your results and discuss how the size distributions are different. (10%)

For both room air and outdoor air, the number concentration decrease as particle size increases. The number concentration in outside is higher than that in room air. The possible reason is anthropogenic sources such as construction and combustion. The filtered air through HVAC system goes into the building (room).
Homework IV – Coagulation, Nucleation & Condensation (due April 02, 200)

1. In an experiment using cadmium oxide smoke, the particle concentration was recorded as follows:

<table>
<thead>
<tr>
<th>Time from Start (min)</th>
<th>Number conc. (#/cm$^3$ X 10$^{-6}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.92</td>
</tr>
<tr>
<td>24</td>
<td>0.47</td>
</tr>
<tr>
<td>43</td>
<td>0.33</td>
</tr>
<tr>
<td>62</td>
<td>0.24</td>
</tr>
<tr>
<td>84</td>
<td>0.21</td>
</tr>
</tbody>
</table>

It is argued that the major mechanism in this system was Brownian coagulation only. Determine the coagulation constant from these data and compare with theory for monodisperse aerosols. (10%) 

Sol>

The graph shows a strong linear correlation ($K_0=8.2\times10^{-10}$ cm$^3$/s) and a good assumption of the aerosol as nearly monodisperse. The theoretical value is $K_0=3.0\times10^{-10}$ cm$^3$/s. If the particle follows the theory of monodisperse, the $C_c$ might be 2.7 and the particle size is around 0.1µm.
2. A log-normal aerosol model (BIMODALM.EXE) has been developed to help you understand the dynamic behavior of aerosols undergoing coagulation and/or condensation. Instruction of how to run the program is provided in “README.TXT” and “INPUT HELP.DOC”. Use it to study the following scenarios: (a) coagulation only – run for a total time of 10 s (which is equal to the product of time step (dt) and total number of steps (IT)); (b) condensation only – run for a total time of $1 \times 10^{-2}$ s. Use the default values specified in INPUT and choose the uni-modal option. However, for each scenario, choose two $\sigma_g$'s (1 and 2). Plot the number concentration (N), particle diameter ($d_g$), geometric standard deviation ($\sigma_g$) and saturation ratio (S) as a function of time for each scenario. In addition, plot particle size distributions at the beginning and at the end of each scenario. Report your observation and explain why it is so. (20%)

Sol>

Change of number concentration: condensation doesn't change the number of particles while coagulation quickly reduces number concentration of particles.
Change of MMD: for coagulation, when $s_g$ is greater, $d_g$ increases faster because large particles serve as sites and small particles can quickly attack the sites due to diffusion. For condensation, difference is not significant.

Change of $s_g$: for all cases, the final trend is going to constant because small particles all go to larger particles and particle size difference becomes smaller and smaller until almost no difference. For coagulation, when $s_g=1$, $s_g$ increases a little bit before goes to constant; when $s_g=2$, $s_g$ decreases to a certain level and gets stable. For condensation, $s_g=1$, all particles undergo same growth, so $s_g$ is constant; when $s_g=2$, smaller particles evaporate due to kelvin effect and condense on larger particle, so $s_g$ gets smaller until the difference becomes negligible.

Change of saturation: for coagulation, there is no condensation so no saturation change. For condensation, gaseous compounds go to particle phase so saturation decreases.
For coagulation ($s_g=1$), particles collide with each other and grow. The collision is not of same pattern so after a while $s_g$ becomes greater than 1, so monodisperse becomes polydisperse. MMD gets greater and total number concentration gets smaller; For condensation ($s_g=1$), all particles undergo same growth, so finally, only the MMD gets greater, other parameters stay same.
3. It is desired to quench the coagulation of an aerosol in the free molecular regime \((d_p << \lambda)\). Determine the dilution ratio if the coagulation rate is to be reduced to 1% of its original value by isothermal, constant pressure dilution with particle-free gas. If the rate is to be reduced by the same factor by a reversible adiabatic expansion, determine the volume expansion ratio assuming the gas is ideal. (15%)

Sol>

\[ \frac{dN}{dt} = -k_0 N^2 \quad (\text{Eq. 12.6 \& 12.7}) \]

\[ 0.01 \frac{dN}{dt} = -k_0 (0.1N)^2 \]

\[ \text{Dilution factor} \rightarrow 10 \]

b) adiabatic expansion

\[ \left( \frac{T_2}{T_1} \right) = \left( \frac{V_1}{V_2} \right)^{k-1} \quad (\text{Eq. 12.4}) \]

\[ V_1 = \frac{dN_1}{dt} = -\frac{4kT_1c_v}{3N} \left( \frac{N_1}{V_1} \right)^2 = \frac{4kT_1c_v}{3N} \left( \frac{N_2}{V_2} \right)^2 \]

\[ V_2 = \frac{-4kT_2c_v}{3N} \left( \frac{N_2}{V_2} \right)^2 \]

\[ \frac{V_2}{V_1} = \frac{T_2}{T_1} \left( \frac{V_1}{V_2} \right)^{k-1} = \left( \frac{V_1}{V_2} \right)^{k+1} \]

\[ = \left( \frac{V_1}{V_2} \right)^{1.38 + 1} = \left( \frac{1}{100} \right) \]

\[ \left( \frac{V_1}{V_2} \right)^{2.38} = \frac{1}{100} \]

\[ \frac{V_1}{V_2} = 0.147 \]

\[ V_2 = V_1 = 6.81 \pm 1 \]
4. Determine the relative importance of thermal versus gradient (shear) coagulation for the flow of an aerosol \( d_p = 1.0 \, \mu m \) in an 8-in duct at 2000 ft/min in the region 10 \( \mu m \) from the duct wall. The velocity at a distance \( y \) from the duct wall is given by

\[
U(y) = \frac{y f U^2 \rho_s}{2 \mu} \quad \text{for} \quad y \ll d, \quad \text{(duct size)}
\]

where \( f \) is 0.017 for this particular case. (15%)

- thermal versus gradient coagulation

\[
dp = 1.0 \, \mu m
\]

\[
8 \, \text{in duct}
\]

\[
2000 \, \text{ft/min} = 10.16 \, \text{m/s}
\]

\[
U(y) = \frac{y f U^2 \rho_s}{2 \mu} \quad y \ll d, \quad \text{(duct size)}
\]

\[
f = 0.017
\]

\( \text{sol} \) the ratio of the rate of gradient to thermal

\[
= \frac{dp}{6 \pi D} = \frac{dp^2 (\frac{dU}{dy})}{6 \pi D}
\]

\[
= \frac{(1 \times 10^{-6})^2 (\frac{dU}{dy})}{6 \pi \times 2.74 \times 10^{-11}} \quad (D \text{ from Appendix in textbook)}
\]

\[
= 15^{12} \cdot 0.017 (10.16)^2 \times 1.2
\]

\[
6 \pi \times 2.74 \times 10^{-11} \quad 2 \times 1.8 \times 10^{-9}
\]

\[
\geq 10^8
\]

\( \text{Gradient coagulation is important} \)
5. An investigator reports observing an aerosol for 10 min and then measuring the number concentration as $10^7$#/cm$^3$. Is this reasonable? If yes, calculate the original concentration. If not, explain why not. Assume simple monodisperse coagulation with a coagulation coefficient of $3.0 \times 10^{-10}$ cm$^3$/s.

(10%)

Sol>

<table>
<thead>
<tr>
<th>Coagulation Coefficient</th>
<th>$K$</th>
<th>$3 \times 10^{-10}$ cm$^3$/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>$t$</td>
<td>10 min</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>600 sec</td>
</tr>
<tr>
<td>Original Concentration</td>
<td>$N_0$</td>
<td>1.00E+00</td>
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<tr>
<td></td>
<td></td>
<td>1.00E+01</td>
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<td>Final Concentration</td>
<td>$N$</td>
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<td>5.56E+05</td>
</tr>
</tbody>
</table>

This is not possible because as the original concentration increases, the final concentration approaches the value $5.56 \times 10^6$ but never reaches $1 \times 10^7$. The coagulation has asymptotic behavior.
6. Calculate the maximum expansion ratio for adiabatic expansion of saturated water vapor without homogeneous nucleation. Assume that the initial temperature is 20 °C and that a saturation ratio of 4.3 is required for homogeneous nucleation. (10%)

\[
\text{maximum expansion ratio} \\
\text{adiabatic expansion} \\
T_0 \ = \ 20°C \ = \ 293 \ K \\
S_e \ = \ 4.3
\]

\[
S_e = \frac{P_{sat}}{P_e} = \frac{\exp \left( 16.7 - \frac{406}{293-32} \right)}{\exp \left( 16.7 - \frac{406}{293-32} \right)} = 4.3 \quad (\text{Eq. 13.2 \& 13.3})
\]

\[
\frac{16.7 - 406}{293-32} = \ln \left( \frac{2.32}{4.3} \right)
\]

\[T_e = 271 \ K\]

\[
\left( \frac{V}{V_{sat}} \right)^{1.4} = \frac{T_{sat}}{T_e} = \frac{293}{271} = 1.08
\]

\[
\left( \frac{V}{V_{sat}} \right)^{1.6-1} = 1.08
\]

\[
\left( \frac{V}{V_{sat}} \right)^{0.4} = 1.08
\]

\[
\left( \frac{V}{V_{sat}} \right)^{0.8} = 1.08^{0.8} = 1.21
\]

:: maximum expansion ratio : \boxed{1.21}
7. (a) To generate a test aerosol, a suspension of polystyrene spheres in water is aerosolized as 20 μm droplets and dried by mixing with a large volume of air at 60% RH. How much residence time is required after mixing to ensure complete drying of the droplets? T = 20 °C. (b) What change in aerosol mass concentration occurs in a condensation nuclei counter when 0.02 μm salt nuclei at a concentration of 10^4 #/cm^3 grow to 10 μm water droplets? If the volume of the condensation chamber is 240 cm^3, how much water is needed for this process? (10%)

![Equation Image]
b) Mass concentration

\[ P \times \Delta V = 10^5 \times \frac{\pi}{6} \left( (10 \times 10^{-6})^3 - (0.02 \times 10^{-6})^3 \right) \]

\[ = 523.6 \times 10^{-15} \text{ kg} = 5.236 \times 10^{-13} \text{ kg} \]

Total number particle = \( \frac{10^{14} \text{ mol}}{\text{m}^3} \times 2 \times 10^{-6} \text{ m}^3 \)

\[ = 2.4 \times 10^8 \text{ mol} \]

\( (\Delta n)_{\text{S}} = 5.236 \times 10^{-15} \text{ kg} \times 2.14 \times 10^6 \)

\[ = 1.26 \text{ mg} \]
9. Consider a solution droplet that contains a fixed quantity of nonvolatile solute, and allow the amount of solvent in the droplet to vary. For a droplet that contains a large amount of solvent, the equilibrium solvent vapor pressure approaches the vapor pressure of the solvent above a planar surface. As the amount of solvent is reduced, the equilibrium vapor pressure of the solvent passes through a maximum due to the Kelvin effect and then decreases below the planar surface value of the solvent as a result of the high concentration of dissolved solute. (a) Show that the vapor pressure maximum occurs when \( \frac{d_p}{\sqrt{2 \pi \sigma}} = \sqrt{\frac{9 n_2 RT}{2 \pi \sigma}} \) for an ideal solution. In this expression, \( n_2 \) is the (fixed) number of moles of solute dissolved in the droplet. (b) A small aqueous solution droplet of sulfuric acid is in equilibrium with water vapor in air at 25°C. The mass of sulfuric acid in the droplet is \( 10^{-6} \) g. Prepare a figure that shows \( \log \frac{p}{p_{so}} \) as a function of droplet diameter where \( p = \) equilibrium droplet vapor pressure and \( p_{so} = \) equilibrium vapor pressure above a planar surface of the solvent. Diagrams of this type are called Koehler curves. (12%)
10. Determine the size of the smallest stable droplet at the critical saturation ratio for toluene at 300 K. Of how many molecules are these droplets composed? (8%)

\[ \text{toluene } S = 2, \text{ mol K } \]

\[ \sigma = 28.53 \text{ dyn cm}^{-1} = 0.0285 \text{ N m}^{-1} \]

\[ \mu = 0.214 \text{ g mole}^{-1}, \rho = 0.8669 \text{ g mL}^{-1} \quad \text{(from Wikipedia)} \]

\[ \begin{align*}
D_p^* &= \frac{2\sigma \nu_i}{kT \ln s} \\
N_e^* &= \frac{32\pi \sigma^3 \nu_i^2}{3(4\pi \epsilon \sigma)^3} \\
\end{align*} \quad \text{(from class notes)}
\]

\[ \begin{align*}
P_p^* &= \frac{2 \times 0.02853 \times 17.65 \times 10^{-29}}{1.38 \times 10^{-23} \times 360 \times 360} = 3.5 \mu m \\

\nu_i &= \frac{M}{(N\sigma)_u} = \frac{92.14}{0.8669 \times 6.02 \times 10^{23}} = 17.65 \times 10^{-29} \\

N_{e}^* &= \frac{32\pi \times 0.02853^3 \times (17.65 \times 10^{-29})^2}{3(1.38 \times 10^{-23} \times 360 \times 360)^2} \\
\end{align*} \]

\[ \approx 1000 \]